

文章编号:1000-8055(2012)03-0566-06

Analysis of the buoyancy-induced heat transfer in a rotating cavity

TIAN Shu-qing^{1,2}, XUE Hang³, GAO Chao⁴, LI Ying⁵

(1. Technology Department, Aviation Industry Corporation of
China Commercial Aircraft Engine Company Limited, Shanghai 200241, China;

2. Institute of Steam and Gas Turbines,

Rheinisch-Westfaelische Technische Aachen University, Germany;

3. Civil Aircraft Marketing & Sales,

Aviation Industry Corporation of China International Holding Corporation,
Beijing 100101, China;

4. School of Jet Propulsion, Beijing University of Aeronautics and Astronautics,
Beijing 100191, China;

5. Hekou Oil Production Plant of Shengli Oilfield Sinopec Group,
Shandong Dongying 257200, China)

Abstract: In order to get a better knowledge of the heat transfer in compressor cavities of aero-engines, the simplified rotating cavity with two-plane discs, a shaft and a cylindrical rim has been investigated numerically and compared with the available measurements. The numerical results in agreement with the available experiments show large-scale instabilities. The disk local Nusselt numbers show mainly radial rising distributions for the heated disks with radial rising temperature profiles. In the present work, at the Reynolds number of 20 000, the disk local Nusselt numbers are the correlations of the local Grashof number to the power of 1.89~2.6, and the value of the power is increased as the rotational Reynolds number goes up. At the rotational Reynolds number of 800 000, the local Nusselt numbers are the correlations of the local Grashof number to the power of 0.68~2.6, and the value of the power is decreased as Reynolds number goes up. The area-averaged disk Nusselt number is the correlation of the Reynolds number to the power of 0.479 and the rotational Grashof number to the power of 0.12.

Key words: buoyancy force; Coriolis force; rotating cavity; heat transfer; turbine

CLC number: V231.1

Document code: A

Nomenclature

$Nu = qr / \lambda \Delta T$	local Nusselt number	$\Delta T = T_w - T_{in}$	temperature difference between wall and inlet
$Gr = \frac{\rho^2 \Omega^2 r}{\mu^2} l^3 \beta \Delta T$	local Grashof number	ΔT_{max}	maximum temperature difference between wall and inlet
$Gr_\phi = Re_\phi^2 \beta \Delta T_{max}$	maximum Grashof number	c	mean axial velocity at the inlet
$Re = 2\rho c r_i / \mu$	axial Reynolds number	$l = r_0 - \Delta r - r_i$	characteristic length
$Re_\phi = \rho \Omega r_0^2 / \mu$	rotational Reynolds number	q	heat flux
T	temperature	r	radius
T_w	temperature of the wall	r_i	radius of the inner shaft
T_{in}	inlet temperature of the cooling air	r_0	radius of the shroud

Received: 2011-04-20; revision received: 2011-11-01

E-mail: tianshuqing@acaе.сom.сn

Δr	radial gap for inlet and outlet of cooling air	$\bar{V}$	velocity components in Cartesian coordinate
s	disk gap	y	generalized wall distance
t	time	$y^+ = yU_y/\nu$	wall distance Reynolds number
$U_\tau = \sqrt{\tau_w/\rho}$	friction velocity		

Greek letters

λ	heat conductivity	ρ	density
μ	dynamic viscosity	Ω	angular velocity of the cavity
τ_w	total shear stress at the wall		

A great potential for further development of aero-engines as well as land-based gas turbines lies in the improvement of component efficiencies. Consequently, the higher gas temperatures at the turbine inlet and the higher pressure ratio are used to increase the thermal efficiency. In connection with this trend attention has to be paid to the mechanically and thermally highly loaded components. Some cooling air extracted from the compressor is fed through rotating cylindrical cavities to cool certain engine components with high temperatures, as shown in Fig. 1.

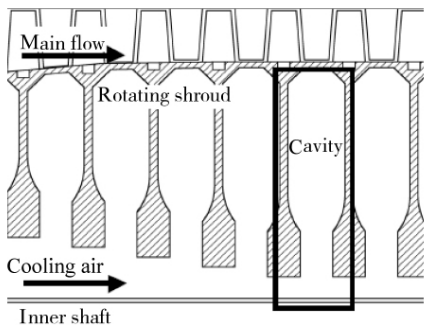


Fig. 1 Rotating cavity with axial through flow in an aero-engine high pressure compressor

Compressor disks can be simplified as two pierced rotating discs in the axial direction and the cylindrical rotating shroud in the outer radial direction and the compressor shaft, which is called a “rotating cavity”. High temperature gradients lead to material stresses, which shorten the fatigue period of the compressor disks. The assurance of the system security requires a detailed understanding of the flow and heat transfer phenomena in the rotating cavities.

A substantial number of relevant experimental and numerical studies have been conducted using idealized plane disk geometries with a

central axial throughflow of cooling air. Farthing P R et al.^[1-2] conducted flow visualization and LDA (laser Doppler anemometry) measurements and showed that the flow patterns and heat transfer in the rotating cavity were functions of the Rossby number and the cavity geometry ratio. They found that the flow in a rotating cavity with heated disks was non-axisymmetric and time-dependent. For the disks heated with radial increasing temperature profiles, the local Nusselt numbers have radial increasing profiles, which were dependent on the local Grashof number with $Gr^{1/4}$, similar to the free convection on a vertical plate under gravity.

Tian et al.^[3-5] computed the flow and heat transfer in a rotating cavity based on the geometry of Farthing P R^[1] et al. Their 3-D(three-dimensional), steady, turbulent computations captured the unstable flow structures with multi-cell circulations. The centrifugal buoyancy force was analyzed to be the key factor that leads to the unstable flow. The heat transfer depends on the Rayleigh number and the Reynolds number.

Sun et al.^[6-7] investigated the centrifugal-driven convection in the rotating cavity numerically and confirmed the large scale unsteady flow structure. In addition, they found that the LES (large eddy simulation) model can achieve better agreement with flow and heat transfer measurements than RANS (Reynolds-average Navier-Stokes) model.

Owen et al.^[8-10] conducted experimental and numerical studies on the rotating cavity. It was reported that the buoyancy induced flow occurs in the rotating cavity with heated walls. The flow is unsteady and 3-D with multi-cell flow structure

thought to comprise of one, two or three pairs of vortices. These flows display the characteristics of self-organizing systems, which are associated with the MEP (maximum entropy production) principle. The most probable macrostates corresponds to the maximum sum of the heat and work transfer to the system. King M P and Wilson M^[11-12] showed a similar flow structure in a rotating annulus cavity with a heated outer cylindrical surface. The computed time-averaged Nusselt numbers are in good agreement with the correlations for Rayleigh-Bénard convection.

Bohn D et al.^[13-15] performed flow visualization and heat transfer experiments and calculations in the rotating cavity with heated disks. These results confirmed the 3-D, unsteady flow. Furthermore, it was reported that the Nusselt numbers at the disks are dependent on the axial Reynolds and Grashof numbers.

This paper presents the numerical investigations for the heat transfer in a rotating cavity with symmetrical heated disks and an axial through flow of cooling air, and compares with the available experiments. The heat transfer with linearly radial rising temperatures towards the shroud at different Reynolds numbers and Grashof numbers are analyzed in detail. The influence of the buoyancy force on the heat transfer is discussed.

1 Numerical algorithm

1.1 Governing equations

The numerical analysis is carried using a commercial CFD (computational fluid dynamics) code, Fluent. The 3-D time-dependent compressible RANS equations were solved in the rotating frame of reference. The governing equations are presented in Eq. (1)~Eq. (3).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{\mathbf{V}}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \bar{\mathbf{V}}) + \nabla \cdot (\rho \bar{\mathbf{V}} \bar{\mathbf{V}}) = -\nabla p + \mu_{\text{eff}} \nabla^2 \bar{\mathbf{V}} - 2\rho \boldsymbol{\Omega} \times \bar{\mathbf{V}} - \rho \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \mathbf{r} \tag{2}$$

$$c_p \frac{\partial}{\partial t}(\rho T) + c_p \nabla \cdot (\rho T \mathbf{V}) = \lambda_{\text{eff}} \nabla^2 T \tag{3}$$

The buoyancy force is performed by the density variation, which is calculated according to the ideal gas law with the constant pressure because of the small pressure variation in these calculations.

The equations governing the fluid motion were solved using FV (finite-volume) method with SIMPLE (semi-implicit method for pressure linked equations) pressure-velocity correction algorithm. The second-order-upwind space-discrete, PRESTO (pressure staggering option) pressure-discrete and second-order implicit time-step were employed.

1.2 Test cases and solution domain

In the experiments with symmetrical heated cavity disks, a co-rotating inner rotor shaft was included, and the approximately linearly radial rising temperature profiles are shown in Fig. 2. The two lines are the linear temperature profiles used in the numerical calculations. The shroud surface is isothermal by a temperature of 378 K for case 1~case 5, and 353 K for case 6~case 13. The inlet temperature is 298 K. The geometrical parameters of the cavity are shown in Table 1 and the dimensionless parameters of the investigated cases are listed in Table 2. Experimental details can be found in Bohn D et al^[13-15].

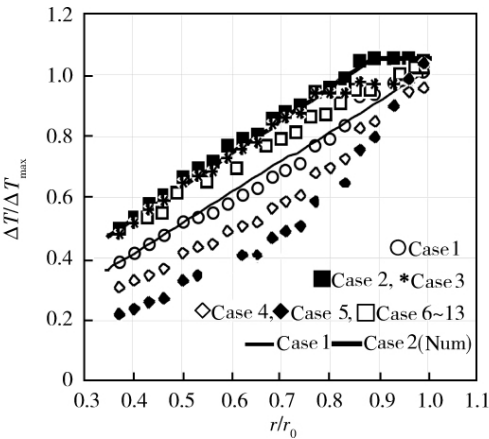


Fig. 2 Radial distribution of $\Delta T/\Delta T_{\text{max}}$ for heated disks^[13-15]

Table 1 Test rig dimension parameters^[14]

s/r_0	r_i/r_0	$\Delta r/r_i$	r_0/m
0.2	0.3	0.15	0.4

The geometry model of the numerical analysis is defined according to the dimensions of the test rig. The sketch of the solution domain and the corresponding computational grid are shown in Fig. 3. The number of the total cells is about 265 000 with 3010 in the r - z plane and 88 in the tangential direction. The grid spacing near walls is refined using geometric expansions, and the closest off-wall node spacing is maintained at $y^+ < 1$.

Table 2 Dimensionless parameters of the investigated cases^[13-15]

Case	$Re/10^4$	$Re_\phi/10^5$	$Gr_\phi/10^{11}$
1	2	2	0.1073
2	2	4	0.4293
3	2	8	0.1717
4	4	8	1.717
5	7	8	1.717
6~13	2~6	10~40	1.845~29.52

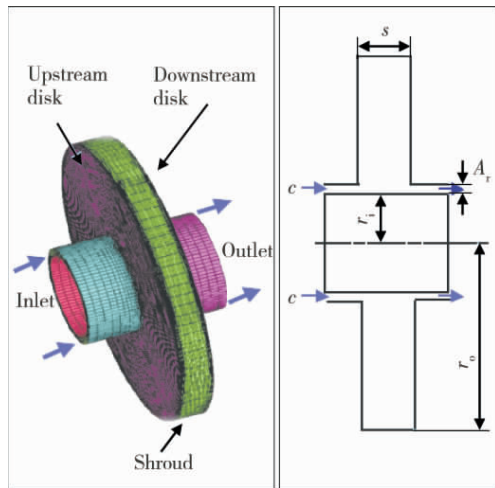


Fig. 3 Sketch of the solution domain and the computational grid

The RNG (re-normalized group) k - ϵ turbulence model was used together with an enhanced wall function model close to the walls. No-slip conditions were used on the walls. A linear profile was used to fit the measured temperature on both disks, a uniform temperature was set on the shroud, and the other surfaces were adiabatic. At the inlet, a uniform velocity condition was used. The pressure condition was defined at the

outlet. The magnitude of the time step was 5 ms for $Re_\phi = 400\,000$ and $Re = 20\,000$.

2 Results analysis

2.1 Numerical results

Fig. 4 shows the heat transfer comparison between the computations and measurements. The measured values of Nu were time averages, and the numerical values were circumferential space averages at certain time step, which were in good agreement with the time averages.

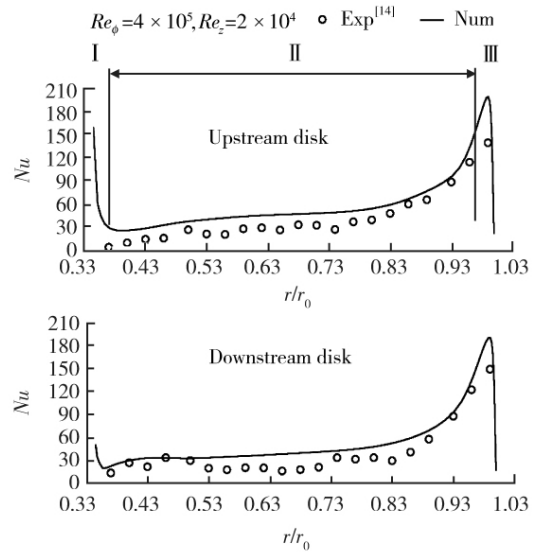


Fig. 4 Computed and measured radial distribution of Nu

The Nusselt numbers on both disks show similar radial distributions, which can be divided into three parts: the near center part I $0.33 < r/r_0 < 0.36$, the near shroud part III $0.96 < r/r_0 < 1$, and the main part II $0.36 < r/r_0 < 0.96$, which is also the main area of the measurements. The computations on both disks in part II are in good agreement with the measurements, the heat transfer goes up with radius rising. In addition, the rising slope of Nu in the region of $0.36 < r/r_0 < 0.8$ is gentle, and in the higher radius region of $r/r_0 > 0.8$, the ascending speed of Nu is fast, which can be explained that the Rayleigh-Bénard convection induced by the centrifugal buoyancy force has more influence on the heat transfer in the higher radius region.

In part I, near the center, the heat transfer is especially high because of the strong forced convection flow, and in part III near the shroud, the heat transfer goes down fast because of the back flow vortices at the corners of disk-shroud. The phenomenon in these two regions is not captured in the experiments.

2.2 Measurements analysis

Fig. 5 shows the radial distributions of Nusselt number at upstream disk at different rotational Reynolds numbers in the region of $Gr_\phi/Re^2=26.825\sim429.25$ and $Re=2\times10^4$. The equations of $f(Nu, Gr)$ are derived from the measurements. In the definition of Gr , the characteristic length of l is $(r_0-\Delta r-r_i)$. It can be seen that the local Nusselt numbers with radius rising profile are dependent on the local Grashof number with $Gr^{1.89\sim2.6}$. As the rotational Reynolds number Re_ϕ goes up, the power of Gr increases, which means that as Re_ϕ increases the heat transfer in higher radius region is increased faster.

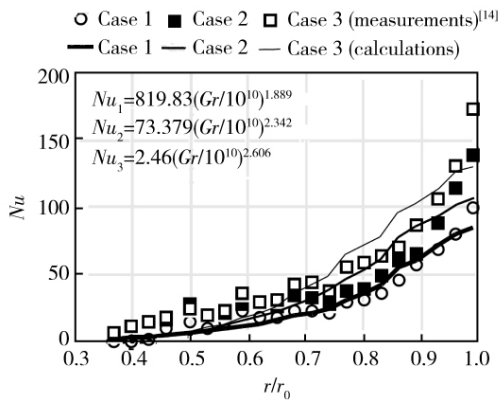


Fig. 5 Nusselt number distributions of upstream disk at $Re=2\times10^4$

Fig. 6 shows the radial distributions of Nusselt number at upstream disk at different inlet Reynolds numbers in the region of $Gr_\phi/Re^2=33.367\sim429.25$. It can be seen that the local Nusselt numbers with radius increasing profile are dependent on the local Grashof number with $Gr^{0.68\sim2.6}$. With the increase of Re , the power of Gr decreases, which can be explained that as the inlet Reynolds number increases, the heat trans-

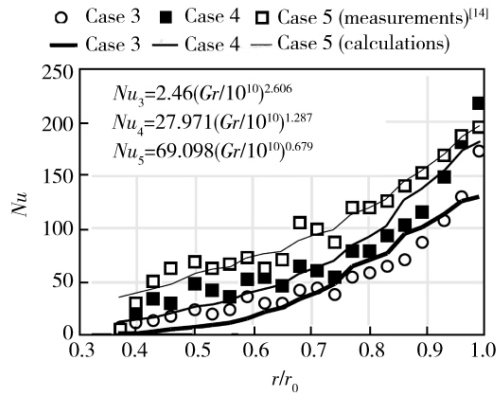


Fig. 6 Nusselt number distributions of upstream disk at $Re_\phi=8\times10^5$

fer in lower radius region is increased larger than that in upper region, thereby the flatter slope of Nu profile.

Based on the analysis of Fig. 5 and Fig. 6, it can be concluded that the heat transfer in the cavity is the combination of the forced convection and the Rayleigh-Bénard convection, the former one has more influence on the Nu in the lower radius region and the latter one has more influence on the Nu in the upper radius region.

Fig. 7 lists the average Nusselt numbers at different parameters, in which the black dots are the calculated data using the Eq. (4), which is derived from the 13 measurements.

$$Nu_{av} = 55.004 \left(\frac{Re}{10^4} \right)^{0.479} \left(\frac{Gr_\phi}{10^{12}} \right)^{0.12} \quad (4)$$

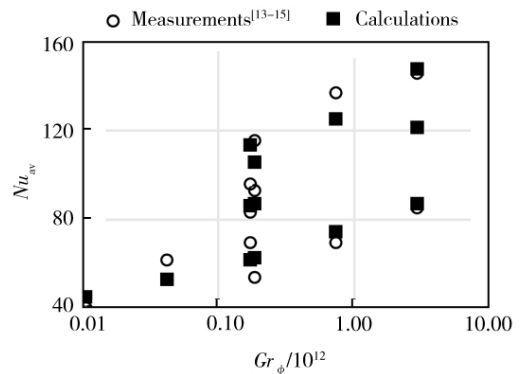


Fig. 7 Variations of the average Nusselt numbers

In Eq. (4), it can be seen that the average Nusselt number is dependent on the Reynolds number with $Re^{0.479}$ and the rotational Grashof number with $Gr_\phi^{0.12}$ in the region of $Re=2\times10^4\sim7\times10^4$ and $Gr_\phi=1.073\times10^{10}\sim2.952\times$

10^{12} . This equation can be used to calculate the heat flux to the fluid, thereby estimate the temperature increase of the outlet fluid.

3 Summary and conclusions

The heat transfer in an annular rotating cavity with cooling air axial through flow and with disks symmetrically heated has been investigated numerically and compared with the experiments.

The computed values of Nu are in reasonably good agreement with the measured data. The local Nusselt numbers at disks show a mainly radius ascending distribution for the disks heated with a radius rising temperature profile. The Rayleigh-Benard convection induced by the centrifugal buoyancy force has more influence on the heat transfer in the upper radii region. In the region of $Re = 2 \times 10^4$ and $Gr_\phi/Re^2 = 26.825 \sim 429.25$, the local Nusselt number are dependent on the local Grashof number with $Gr^{1.89 \sim 2.6}$. As Re_ϕ goes up, the power of Gr increases. In the region of $Re_\phi = 8 \times 10^5$ and $Gr_\phi/Re^2 = 33.367 \sim 429.25$, the local Nusselt numbers are dependent on the local Grashof number with $Gr^{0.68 \sim 2.6}$. As Re goes up, the power of Gr decreases. The average Nusselt number is dependent on the Reynolds number with $Re^{0.479}$ and the rotational Grashof number with $Gr_\phi^{0.12}$. The derived heat transfer equations can be used to estimate the heat transfer to the fluid.

References:

- [1] Farthing P R, Long C A, Owen J M, et al. Rotating cavity with axial throughflow of cooling air: flow structure [J]. ASME Journal of Turbomachinery, 1992, 114 (1): 237-246.
- [2] Farthing P R, Long C A, Owen J M, et al. Rotating cavity with axial throughflow of cooling air: heat transfer [J]. ASME Journal of Turbomachinery, 1992, 114 (1): 229-236.
- [3] TIAN Shuqing, TAO Zhi, DING Shuiting, et al. Investigation of flow and heat transfer in a rotating cavity with axial throughflow of cooling air [R]. ASME Paper GT2004-53525, 2004.
- [4] TIAN Shuqing, TAO Zhi, DING Shuiting, et al. Computation of buoyancy-induced flow in a heated rotating cavity with an axial throughflow of cooling air [J]. International Journal of Heat and Mass Transfer, 2008, 51 (3-4): 960-968.
- [5] TIAN Shuqing, TAO Zhi, DING Shuiting, et al. Numerical investigation on heat transfer in a rotating cavity with axial throughflow of cooling air [J]. Journal of Aerospace Power, 2005, 20(3): 393-396 (in Chinese).
- [6] SUN Zixiang, Kifoil A, Chew J, et al. Numerical simulation of natural convection instationary and rotating cavities [R]. ASME Paper GT2004-53528, 2004.
- [7] SUN Zixiang, Lindblad K, Chew W. LES and RANS investigations into buoyancy-affected convection in a rotating cavity with a central axial throughflow [R]. ASME Paper GT2006-90251, 2006.
- [8] Owen J M, Powell J. Buoyancy-induced flow in a heated rotating cavity [R]. ASME Paper GT2004-3210, 2004.
- [9] Owen J M, Abrahamsson H, Lindblad K. Buoyancy-induced flow in open rotating cavities [R]. ASME Paper GT2006-91134, 2006.
- [10] Owen J M. Thermodynamics analysis of buoyancy-induced flow in rotating cavities [R]. ASME Paper GT2007-27387, 2007.
- [11] King M P, Wilson M, Owen J M. Rayleigh-Benard convection in open and closed rotating cavities [J]. Journal of Engineering for Gas Turbines and Power, 2007, 129 (2): 305-311.
- [12] King M, Wilson M. Free convective heat transfer within rotating annuli [C] // Proc. 12th Int. Heat Transfer Conf. Grenoble, France: University of Bath, 2002: 465-470.
- [13] Bohn D, Bouzidi F, Kitanina E, et al. Numerical and experimental investigations of the air flow and heat transfer in rotating cavities [C] // The 9th International Symposium on Transport Phenomena and dynamics of Rotating Machinery. Honolulu, Hawaii: [s. n.], 2002: 10-14.
- [14] Bohn D, Deutsch G, Simon B, et al. Flow visualization in a rotating cavity with axial throughflow [R]. ASME Paper 2000-GT-280, 2000.
- [15] Bohn D, Deutsch G, Simon B, et al. Visualization of flow phenomena in rotating cavities with axial throughflow and comparison with numerical results [C] // Proceedings of PSFVIP-2. Honolulu, USA: [s. n.], 1999: PF032.